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HYDROGASDYNAMICS IN TECHNOLOGICAL PROCESSES

DESIGN OF MULTIMODE AXISYMMETRIC HYPERSONIC NOZZLES WITH THE USE OF OPTIMIZATION METHODS

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On the basis of the solution of direct problems with the use of various medium models and numerical methods of integration of gas flow equations, methods for designing super- and hypersonic nozzles have been developed. Multimode nozzles in the ranges of Mach numbers 8–14 and 14–20 satisfying given conditions have been designed.

Keywords: *hypersonic nozzle, Navier–Stokes equations, Euler equations, method of characteristics, numerical optimization.*

Introduction. It is known that good quality of flow in a wind tunnel is provided by the application of thoroughly shaped nozzles accelerating the working gas to a given velocity. Such nozzles have a spatial structure, are made with a high precision, and are rather expensive. Practically in all existing wind tunnels single-mode nozzles designed for definite operating conditions are used. Because of the complex structure of single-mode nozzles and the high cost of their production, the number of modes of operation of a wind tunnel is always limited. There exist two approaches to the design of multimode shaped nozzles in which the main outlet section has a fixed geometry and the small separable part adjoining the minimal section makes it possible to vary the Mach number at the outlet from the nozzle. The first approach is based on constructing the nozzle contour with the use of the method of characteristics [1–4], and the second approach is based on methods of direct numerical optimization. For example, in [5] the functional equal to the sum of standard deviations of the Mach number from its average values in a given region of the flow was minimized. Both approaches did not take into account the presence of viscosity and, accordingly, of the boundary layer that radically changes the characteristics of hypersonic nozzles [6]. Therefore, to solve the design problem stated, we used a complex approach combining the method of characteristics that permits calculating fairly easily supersonic nozzles with a uniform outlet characteristic for a perfect nonviscous non-heat-conducting gas, direct methods based on the calculation of viscous flows, and methods of numerical optimization.

Construction of the Initial Approximation. The first stage of designing a nozzle consists of constructing by the method of characteristics the nozzle contour for the maximum value of the Mach number at the outlet and determining the size of the acceleration part of the nozzle for the minimum Mach number. This makes it possible to divide the contour into a fixed equailizing part and a variable accelerating part (much smaller in size) whose shape is determined for the chosen value of the Mach number at the outlet from the nozzle [7]. This stage is an important element of the chosen strategy since the solution of optimization problems for viscous flows required large computational resources and is successful, as a rule, only given a good initial approximation.

Design of Multimode Nozzles. Since the viscous flow in a hypersonic nozzle of a given geometry was supposed to be calculated by means of the Fluent package, it was necessary to test the accuracy of the solution of the Navier–Stokes equation with the k–ω-model of turbulence. The contour obtained by the method of [5] was used in creating a real axisymmetric nozzle with a nozzle outlet diameter of 0.40 m meant for obtaining Mach numbers 8, 10, 12, and 14 at the outlet. The comparison between the experimental and numerical distributions of the Mach number

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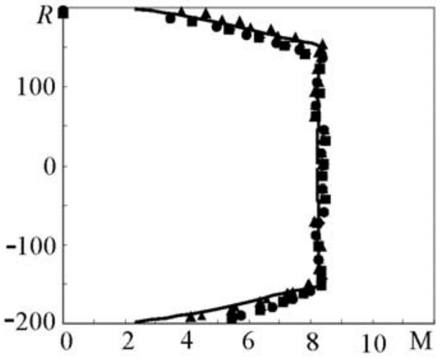

Fig. 1. Comparison between the calculated and experimental distributions of the Mach number on the nozzle exit section (nozzle for M = 8); dots represent the experimental data obtained in a series of starts of the nozzle.

TABLE 1. Standard deviation ΔM from the average Mach number and average Mach number in the section x = 4.0 m

Mesh	ΔM, %	M_{av}
1000 × 40	0.17	8.10
2000 × 40	0.15	8.10
1000 × 80	0.17	8.11

over the nozzle exit section for M = 8 is shown in Fig. 1. The technology of performing experiments is described in [8]. The computational mesh had 1000 nodes along the nozzle axis and 40 nodes along its radius. To test the convergence of the solution, we performed calculations with node doubling. Table 1 presents the standard deviation from the average Mach number and the average Mach number in the section x = 4.0 for M = 8 obtained in using various computational meshes. It is seen that the doubling of the number of nodes in each direction practically does not affect the nozzle characteristics used in calculating the minimized functional of the optimization problem. Therefore, for further calculations we chose the least possible number of nodes, which is essential for decreasing costs in solving optimization problems.

On the basis of the developed technology, we designed a new multimode nozzle with outlet Mach numbers 8, 10, 12, 14. To construct the main contour for M = 14, we considered, as the initial nozzle, the nozzle constructed by the method of characteristics for the uniform Mach number M = 1.57 at the outlet. The obtained contour, given in tabular form with a large number of points (up to 600), was approximated by a cubic spline on a small (10) number of nonuniformly arranged reference points with a mean absolute and relative error of $\sim 10^{-4}$. This is essential so that one can vary the contour in solving optimization problems with the use of a small number of parameters, which will make it possible to decrease the costs in calculating the viscous flow in the nozzle.

Then we modified the initial contour by solving the optimization problem. The viscous flow in optimizing and designing was calculated with the aid of the Fluent package but already integrated with the package of optimization programs. The functional equal to the sum of standard deviations of the Mach number from its average value in a given region of the flow was minimized. To minimize the functional, we used nongradient search methods with adaptation. Restrictions are taken into account by the method of penalty functions. Modification of the basic contour is necessary since calculations of the flow with account for viscosity in hypersonic nozzles designed by the method of characteristics point to the probability of appreciable oscillations of parameters along the nozzle axis [7]. Figure 2 shows the obtained distribution of the Mach number along the axis for the initial and modified nozzles. It is seen that, in the initial nozzle on the axis, oscillations of the Mach number appeared that were not present in the nonviscous flow, and in the modified flow they markedly decreased. Figure 3 compares the calculated and experimental distribution of the Mach numbers at distance x = 0.4 m from the nozzle exit section (a) and on the nozzle exit section (b) for M = 14.

Next we calculated the coordinates of the contour of separable nozzle sections that provide Mach values of 8, 10, 12 at the nozzle outlet. To this end, we solved the optimization problem, in which at a fixed length of the nozzle

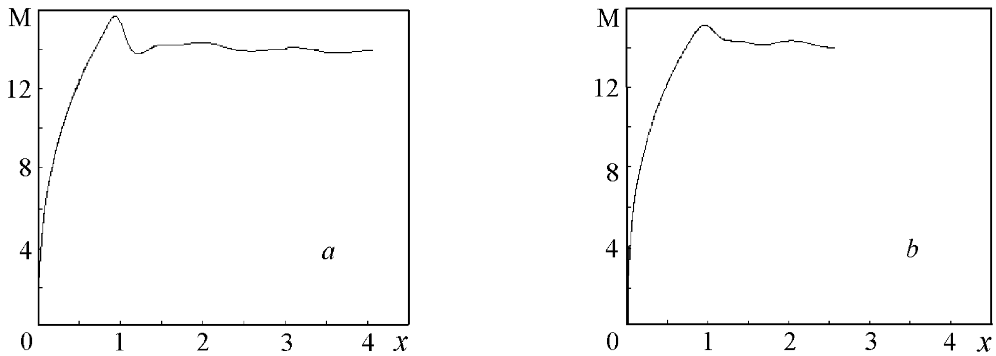

Fig. 2. Mach number distribution along the axis for the initial (a) and modified (b) nozzles.

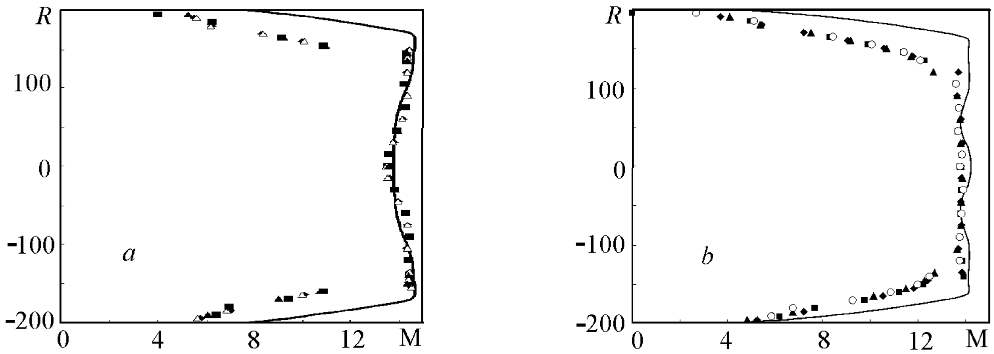

Fig. 3. Comparison between the calculated and experimental distributions of the Mach numbers at a distance x = 0.4 m from the nozzle exit section (a) and on the nozzle exit section (b) (nozzle for M = 14); dots represent the experimental data obtained in a series of starts of the nozzle.

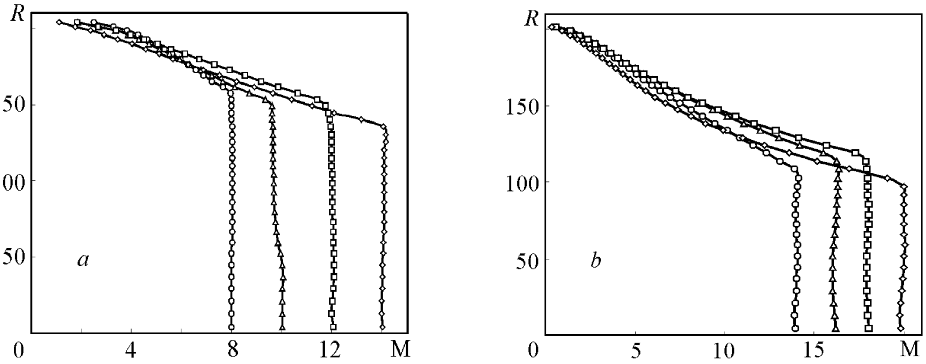

Fig. 4. Calculated distribution of the Mach number on the nozzle exit section. Multimode nozzle for Mach numbers 8, 10, 12, 14 (a) and 14, 16, 18, 20 (b). Curves — from left to right.

of 3 m and an output section diameter of 0.41 m for a given M the critical section diameter and the geometry of the initial part of length 0.32 m were varied. The geometry of the initial part was given by a cubic spline on a nonuniform grid and mated smoothly with the fixed part of the nozzle corresponding to the optimal nozzle for M = 14. The whole of the nozzle was designed for the average values of Reynolds number $\mathrm{Re}_{\mathrm{av}}$ proceeding from the possible parameters of the settling chamber gas. Figure 4a shows the Mach number distribution in the obtained nozzles for all calculated regimes in the section x = 3 m. The uniformity characteristics of the obtained flow in the nozzle are given in Table 2. It is seen that except for the variant M = 10 uniformity of the flow in the working zone of the nozzle may be expected to constitute 0.3–0.4%. It is important to know the characteristics of designed nozzles for the mini-

TABLE 2. Standard deviation (%) from the average Mach number in the section and on the intercept. Nozzles for Mach numbers 8, 10, 12, 14

x, m	M			
	8	10	12	14
2.5	0.25	1.47	0.58	0.41
3.0	0.16	1.68	0.29	0.29
2.5–3.5	0.65	0.40	0.45	0.02

TABLE 3. Standard deviations from the average Mach number and average Mach number in the section and on the intercept for various Mach numbers

Re	x, m					
	2.5		3.0		2.5–3.5	
	ΔM, %	M_{av}	ΔM, %	M_{av}	ΔM, %	M_{av}
Nozzle for M = 8						
$Re_{min} = 9.4 \cdot 10^6$	0.15	7.85	0.12	7.89	0.63	7.91
$Re_{av} = 3.92 \cdot 10^7$	0.25	7.96	0.16	8.01	0.65	7.99
$Re_{max} = 11.9 \cdot 10^7$	0.18	7.85	0.12	7.89	0.29	7.87
Nozzle for M = 12						
$Re_{min} = 2.5 \cdot 10^6$	0.25	11.50	0.49	11.69	0.38	11.60
$Re_{av} = 8.7 \cdot 10^6$	0.58	11.83	0.29	12.03	0.43	12.03
$Re_{max} = 2.35 \cdot 10^7$	0.59	9.820	0.36	10.02	0.53	10.02

TABLE 4. Standard deviation (%) from the average Mach number in the sections and on the intercept. Nozzles for Mach numbers 8, 10, 12, 14

x, m	M			
	14	16	18	20
3.4	0.69	0.60	0.77	0.32
3.9	0.39	0.17	0.70	0.29
3.4–4.4	0.43	0.67	0.84	0.49

mum and maximum values of the Reynolds number of each range. Examples of the results of these calculations for Mach numbers 8 and 12 and the characteristics of the base variants are given in Table 3.

On the basis of the developed design technology, we also constructed a second multimode nozzle with outlet Mach numbers 14, 16, 18, and 20. The initial contour was constructed by the method of characteristics for M = 24. Upon its modification we solved the optimization problem in which at a fixed length of the nozzle of 3.9 m and an outlet section diameter of 0.4 m for a given M only the geometry of the initial part of length 0.32 m was varied. Figure 4b shows the distribution of the Mach number for all the calculated regimes in the outlet nozzle section at x = 3.9 m. It is seen that the boundary layer occupies a fairly large part of the nozzle section.

The uniformity characteristics of the flow obtained are given in Table 4. It is seen that in a wide range of operating conditions uniformity of the flow in the working zone of the nozzle may be expected to constitute 0.3–0.8%.

Conclusions. We have developed a computing technology that permits solving problems of designing and optimizing super- and hypersonic nozzles. It includes the method of characteristics, the direct method, various models of the medium, and numerical methods of integration of viscous gas flow equations. The classes of functions giving the nozzle contour geometry and the space dimension of variable parameters have been chosen. The formulated restrictions, the functionals used, and the solution methods for the minimization problem permit obtaining appropriate solutions of design problems for multimode nozzles in a wide range of requirements placed on them. Two multimode nozzles for Mach numbers 8–14 and 14–20 satisfying the given restrictions have been designed.

NOTATION

k, kinetic energy density of turbulence; M, Mach number at the nozzle outlet; R, nozzle radius; $\mathrm{Re}_{\mathrm{av}}$, $\mathrm{Re}_{\mathrm{min}}$, $\mathrm{Re}_{\mathrm{max}}$, average, minimum, and maximum values of the Reynolds number at the nozzle outlet; x, symmetry axis; ω, value of the dissipation rate per unit kinetic energy of turbulence. Subscripts: av, average; min, minimum; max, maximum.

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